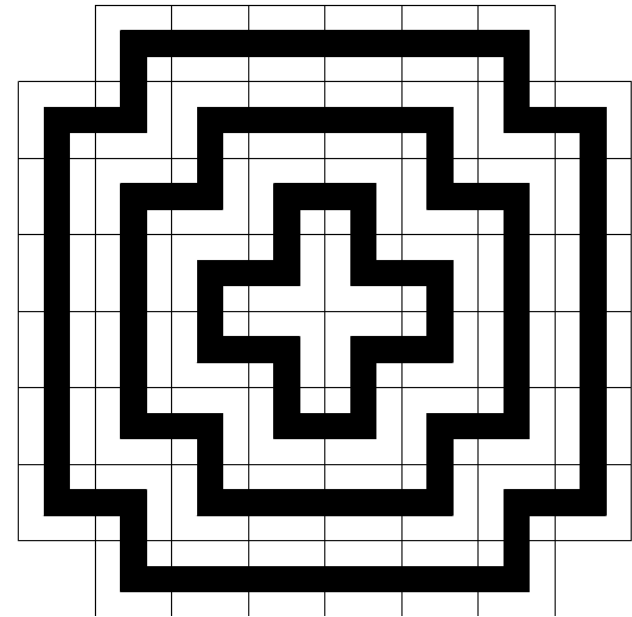


# Track Tiles



**Ideas for teachers**

Ian Sugarman

# Track Tiles

## CONTENTS

<b>3</b>	Why Track Tiles?
<b>4</b>	Constructing Circuits
<b>5</b>	Growing Patterns 1
<b>6</b>	Growing Patterns 2
<b>7</b>	Growing Patterns 3
<b>8</b>	Growing Patterns 4
<b>9</b>	Forming Letters and Numerals
<b>10</b>	The Greek Key Pattern
<b>11</b>	From Here to There
<b>12</b>	Two Tiles Investigation
<b>13</b>	Traversability

<b>14</b>	Creating Pattern 1 – <i>Reflecting</i>
<b>15</b>	Creating Pattern 2 – <i>Translating/Reflecting/ Rotating</i>
<b>16</b>	Activity Sheets for creating reflecting patterns
<b>19</b>	Extended Patterns from Reflective symmetry
<b>21</b>	Photos of Tiles in use
<b>22</b>	Recording sheets
<b>23</b>	Tiles for Photocopying

## Why Track Tiles?

There has been a long held principle that working with structured mathematical equipment is an ideal means of engaging students with mathematical ideas. The practical nature of engaging with the tiles allows you to see more clearly the structural nature of geometric shapes and develop a sense of what underlies the construction of simple algebraic patterns and relationships.

The manipulation of the tiles can also facilitate the development of systematic and logical approaches to problem solving - key elements of mathematical thinking.

Activities with Track tiles provide excellent opportunities for engaging students with all three strands of Using and Applying Mathematics, (**Problem solving, Communicating and Reasoning**),

especially these aspects at Key Stage 1:

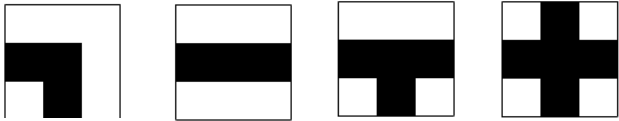
- ***Try different approaches and find ways of overcoming difficulties when solving shape and space problems***
- ***Recognise simple spatial patterns and relationships and make predictions about them***

and at Key Stage 2:

- ***Approach spatial problems flexibly, including trying alternative approaches to overcome difficulties***
- ***Organise work and record or represent it in a variety of ways when presenting solutions to geometrical problems***
- ***Recognise, represent and interpret simple number relationships, constructing and using formulae in words, then symbols***
- ***Use mathematical reasoning to explain features of shape and space.***
- ***Present and interpret solutions to problems***

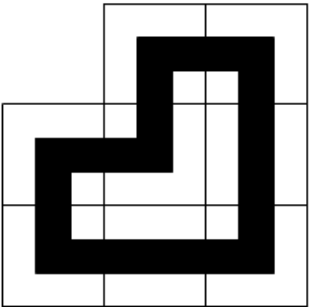
# CONSTRUCTING CIRCUITS

Can you arrange a given set of tiles to make a completed circuit? – no loose ends!

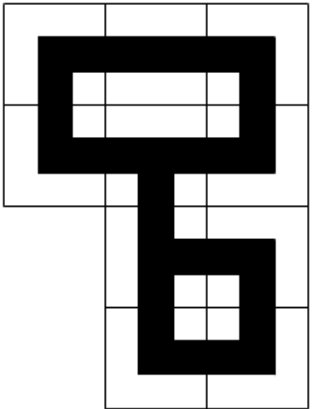
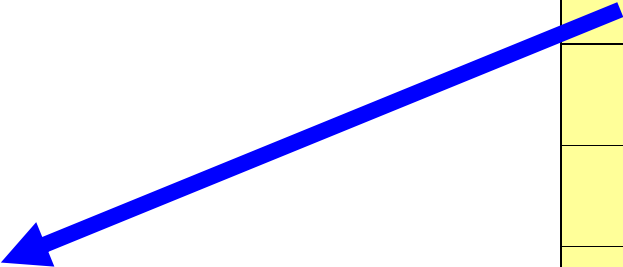


Total

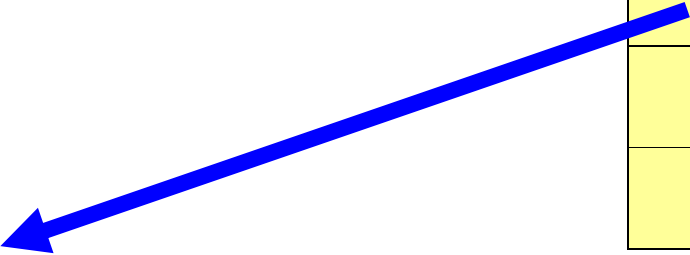
A	6	2	0	0	8
B	5	1	2	0	8
C	6	2	2	0	10
D	6	3	2	0	11
E	5	2	3	0	10
F	7	1	2	0	10
G	4	3	4	1	12
H	5	2	2	1	10



A



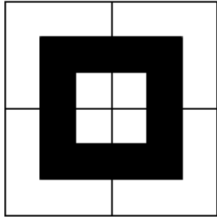
This is only one of six solutions



I could invent a puzzle of my own for a friend to solve

# GROWING PATTERNS 1

Starting from the instruction  
“make the smallest possible  
square”,

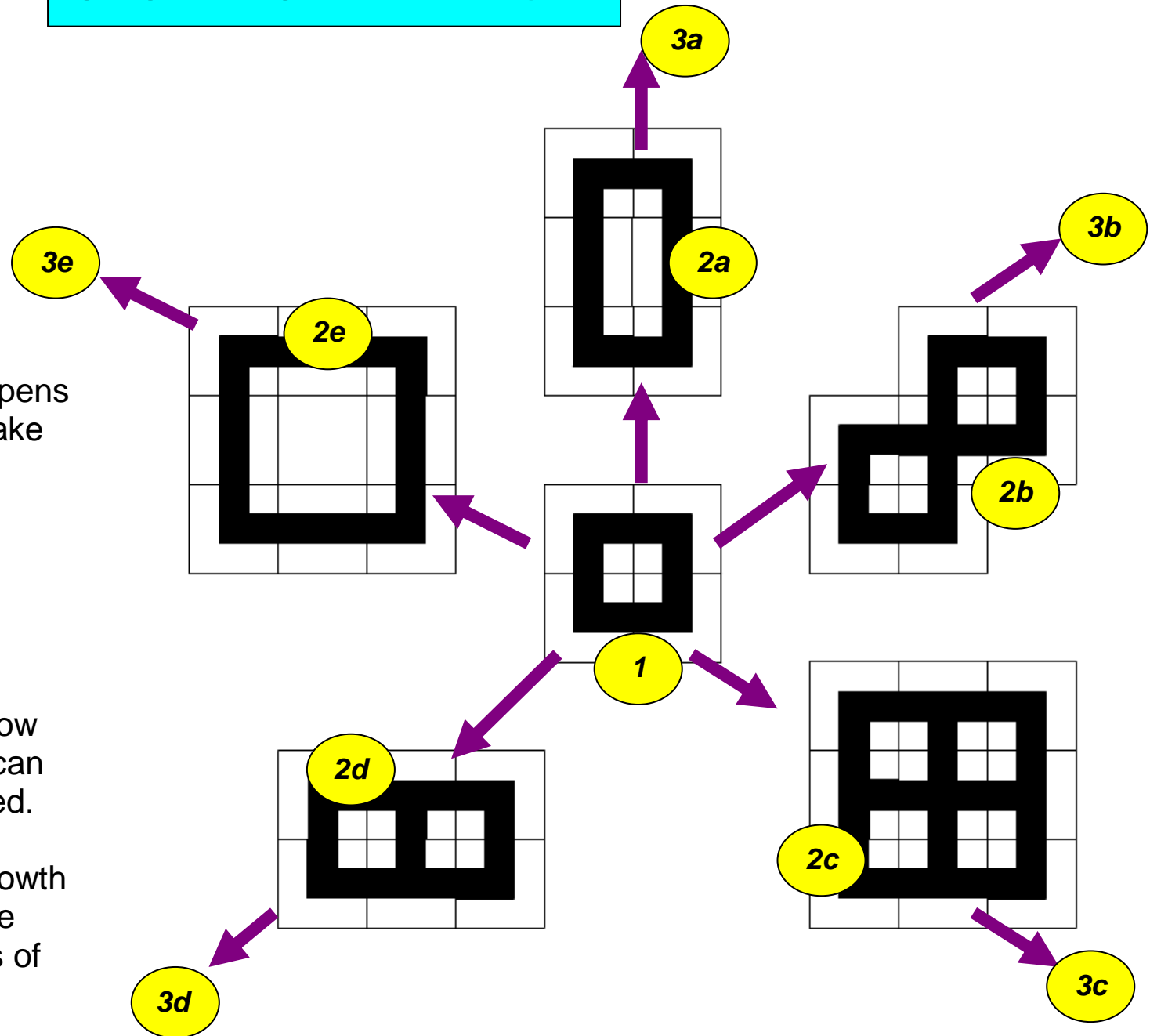


we can investigate what happens  
when we suggest that we make  
this picture “grow”.

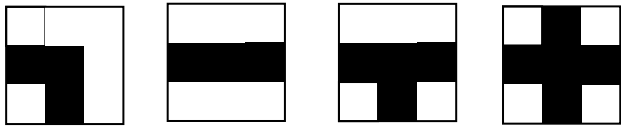
The illustration shows five  
possible responses to that  
suggestion.

Each one of these can be  
adopted as the model to follow  
and the 3<sup>rd</sup> and 4<sup>th</sup> pictures can  
be visualised and constructed.

As each new stage in the growth  
is visualised, a record can be  
kept of the way the numbers of  
tiles grows.....



# GROWING PATTERNS 2



Total

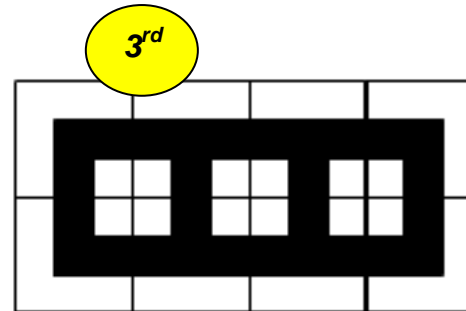
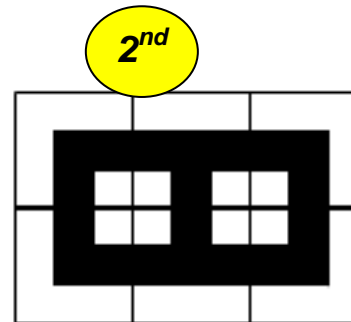
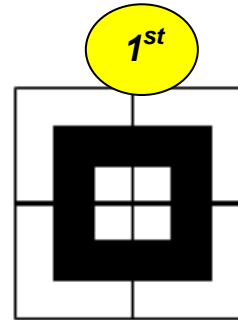
1	4	0	0	0	4
2	4	0	2	0	6
3	4	0	4	0	8
4	4	0	6	0	10
5					

When the table is filled with the results, a clear pattern emerges which can be described as an **adding 2** pattern and be explained by referring to the changes that take place in the way the tiling pattern grows (*an extra 2 "T"s*).

The patterns can also be highlighted in the hundred square.

Seeing the pattern represented in this way allows a range of questions to be answered by interpreting the diagrams....

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



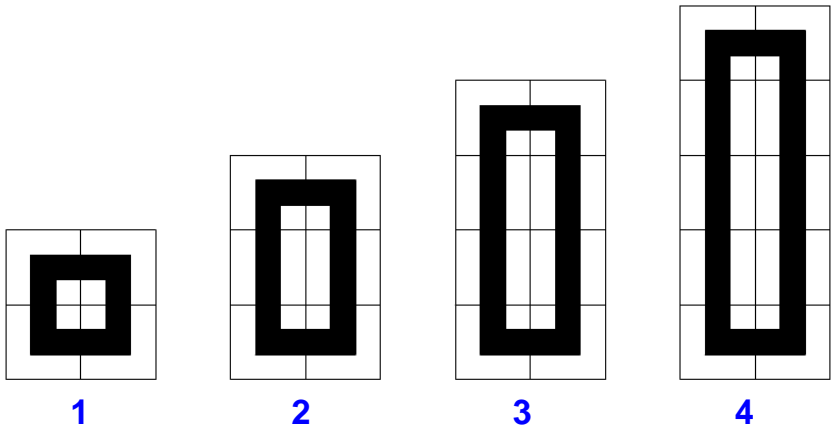
*Can you use the hundred square to tell me how many tiles would be needed to make the 30<sup>th</sup> picture?*

*What if you made the complete set of pictures from the 1<sup>st</sup> to the 10<sup>th</sup>, how many tiles of each kind would you need?*

*Tell me about the 5<sup>th</sup> tiling picture*

*Tell me about the 7<sup>th</sup> tiling picture*

# GROWING PATTERNS 3



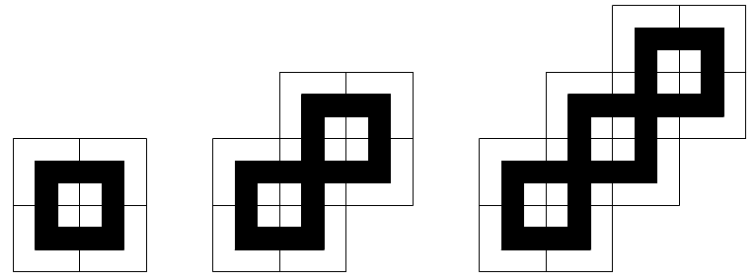
$p$			Total
1	4	0	4
2	4	2	6
3	4	4	8
4	4	6	10

Identifying the growing pattern here is simple and easy to calculate the next few in the series.

But what if the question of the 12<sup>th</sup> or 50<sup>th</sup> picture in the series is asked for?

Attention needs to be focused on explaining how the shape grows in general, rather than in a way that refers to any particular picture e.g. “there are **always** 4 corner tiles, and the number of straight line tiles on each side is **always** one less than the picture no.”

$$4 + [\text{picture number } (p) - 1] \times 2 \rightarrow 2(p - 1) + 4 \rightarrow 2p + 2$$



$p$			Total
1	4	0	4
2	6	1	7
3	8	2	10
4	10	3	13

What may be noticed here is that with each new square, 3 more tiles are needed, one of which needs to be a cross.

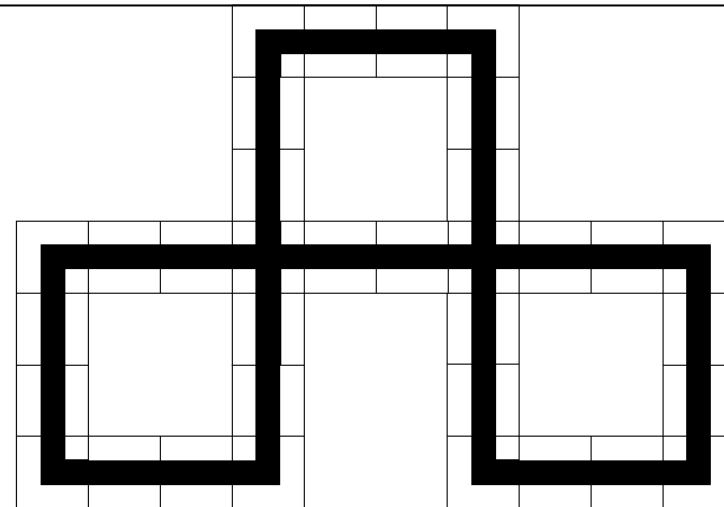
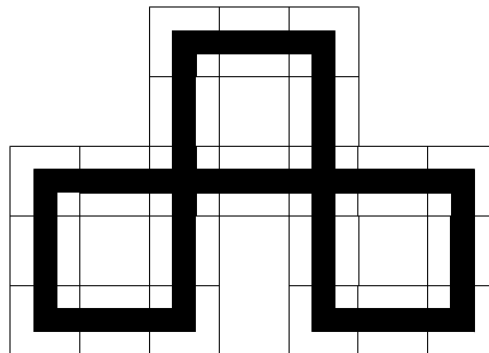
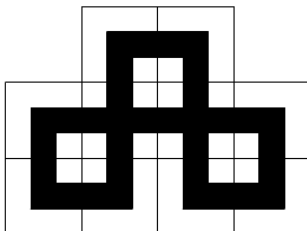
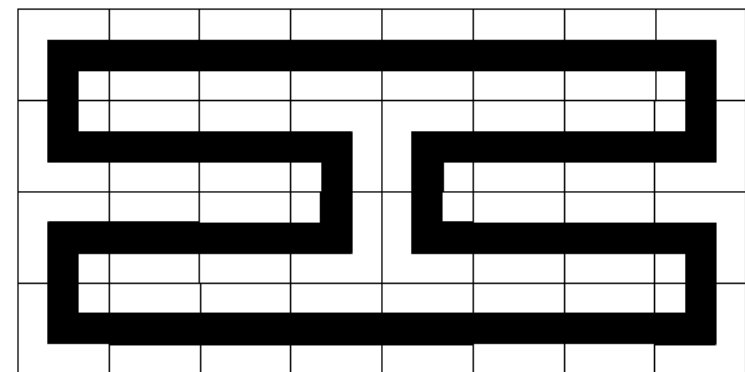
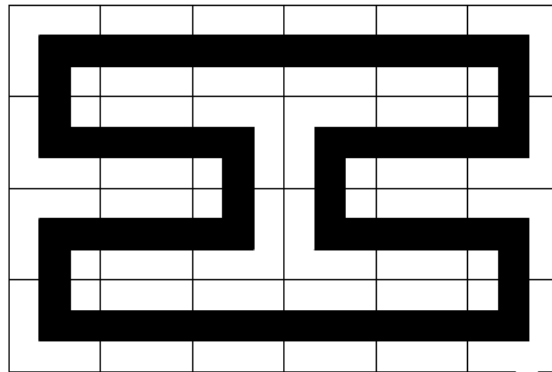
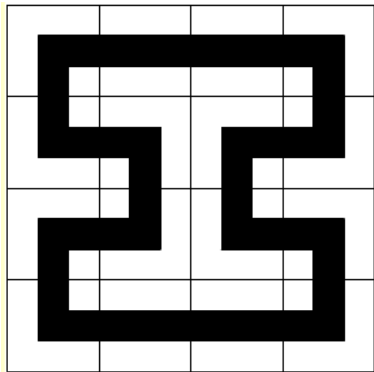
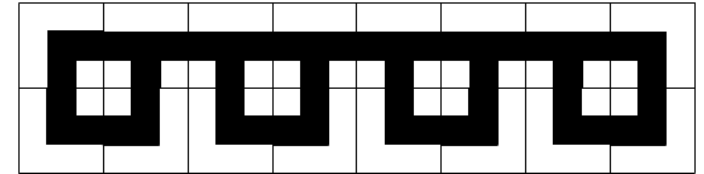
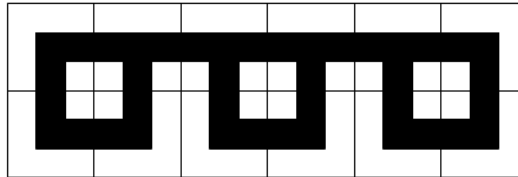
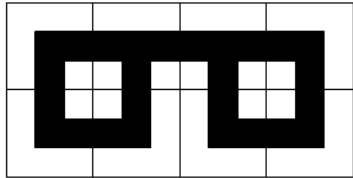
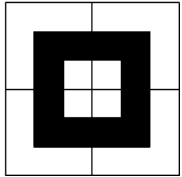
But to predict for a larger number of squares, it is necessary to look for a way of relating the number of cross tiles to the number of squares, however many squares there are!

It might be .....” *the no. of corner tiles on each side is always the same as the number of squares. There are always 2 other corners (top and bottom) and the no. of cross tiles is 1 always less than the number of squares.*

$$2p + 2 + p - 1 \rightarrow 3p + 1$$

# GROWING PATTERNS 4

Investigate the growth in these pictures.  
Can you predict how many of each tile would be needed for Picture number 10 in each series?



# FORMING LETTERS AND NUMERALS

one

0 1 2 3 4 5 6 7 8 9

two

three

FOUR

What's the biggest number you can make with 75 tiles?

What do you mean by "big"?

	3p
	2p
	5p
	10p

What's the cheapest (or dearest) number (or letter)?

TWO

# THE GREEK KEY PATTERN



These Key patterns first appeared in Ancient Greece but were also used by the Romans as border patterns to decorate ornaments and mosaic floors.



a unit

*How many of each tile are needed to create each unit?  
How many repeats of that unit are needed to reach across a sheet of paper? The table? The wall?  
So how many tiles is that?*

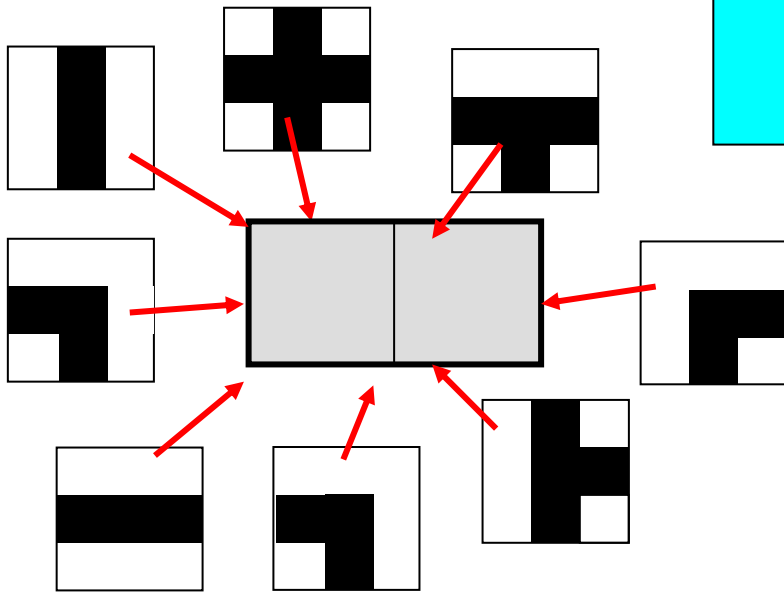
Only these tiles are needed for this pattern

What about distortions of the basic design?

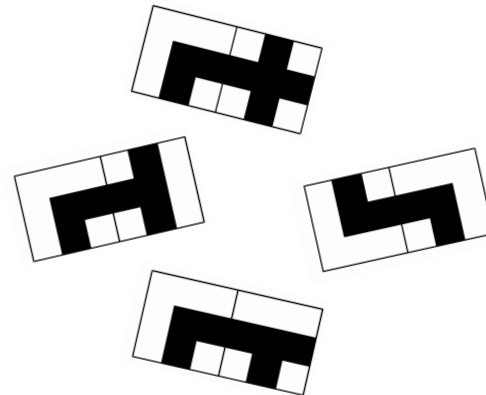
*Can you make the pattern turn a corner?  
Tricky!*



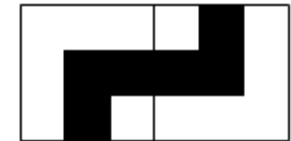
# TWO TILES INVESTIGATION



Place two tiles side by side.  
What different pictures can you make?



Being systematic.....



The left tile is kept as a corner in the same orientation. The right tile is rotated in its 4 positions.

Only two more pairings are possible with 2 corners

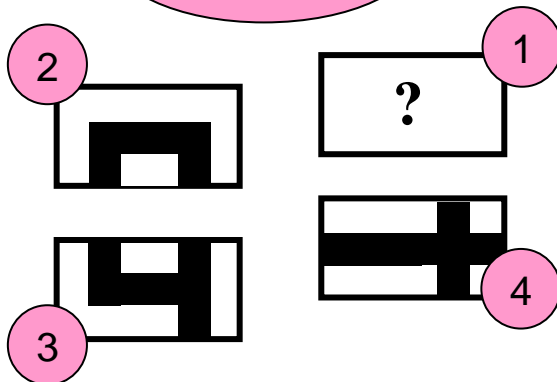


But two of these are really **duplicates** – one is a reflection of the other.

Can you.....

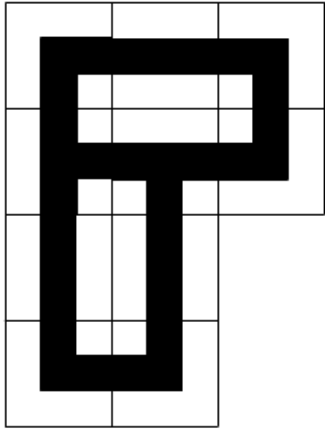
- discover how many **different orientations** each tile has got?
- recognise duplicates when they are in different orientations?
- adopt a systematic approach to be sure not to miss any permutation

How many regions?

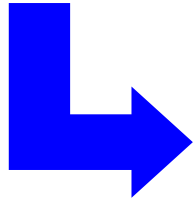
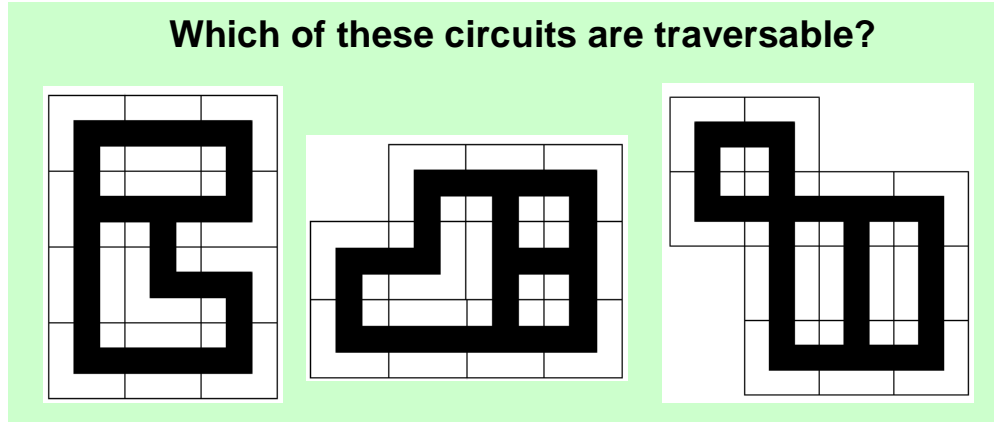


# TRAVERSABILITY

Take any of the circuits...



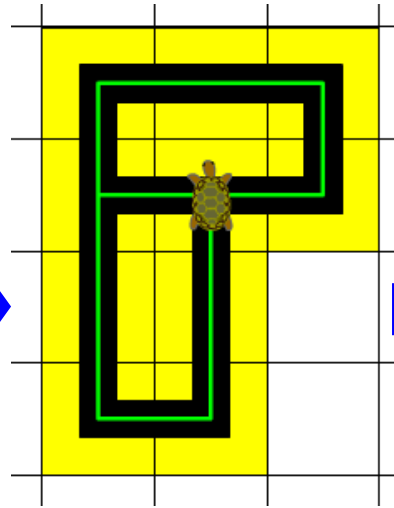
Program a turtle to walk the pathway, but not along the same bit more than once.....



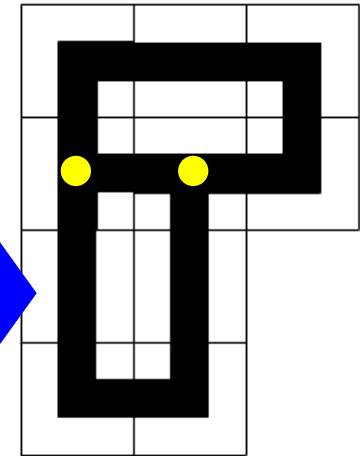
Welcome to the Number Gym

### Route Tiles: Pathways

	A	B	C	D	E	F	
6							6
5							5
4							4
3							3
2							2
1							1
	A	B	C	D	E	F	



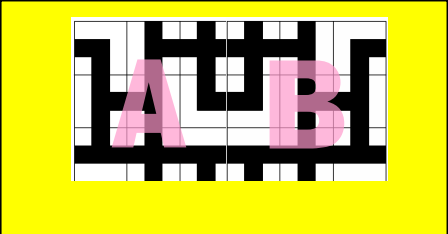
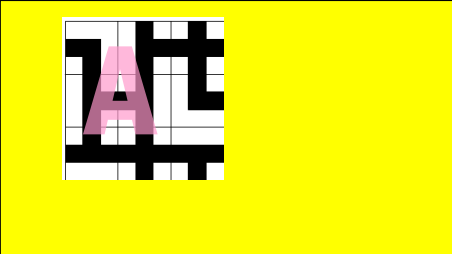
This circuit is traversable, but only from two points



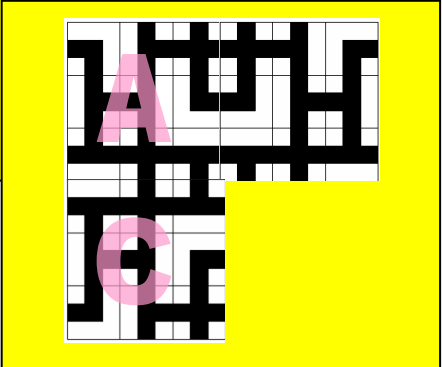
**F,F,R,F,F,F,R,F,F,R,F,F,F,F,L,F,F,F,F,L,F,F,L,F,F,F,F**

Forward, Forward, Right, Forward ....

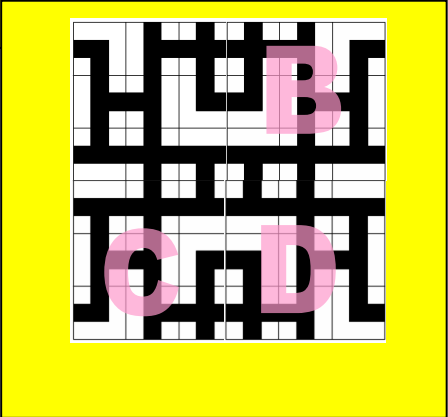
# CREATING PATTERN 1 –Reflecting



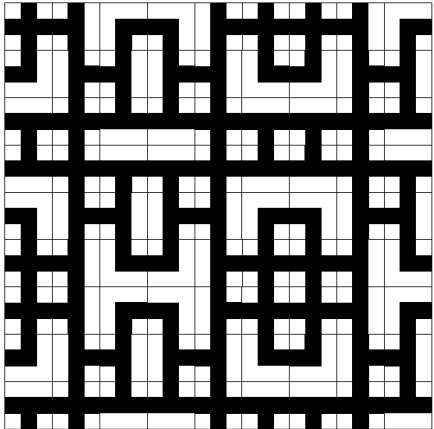
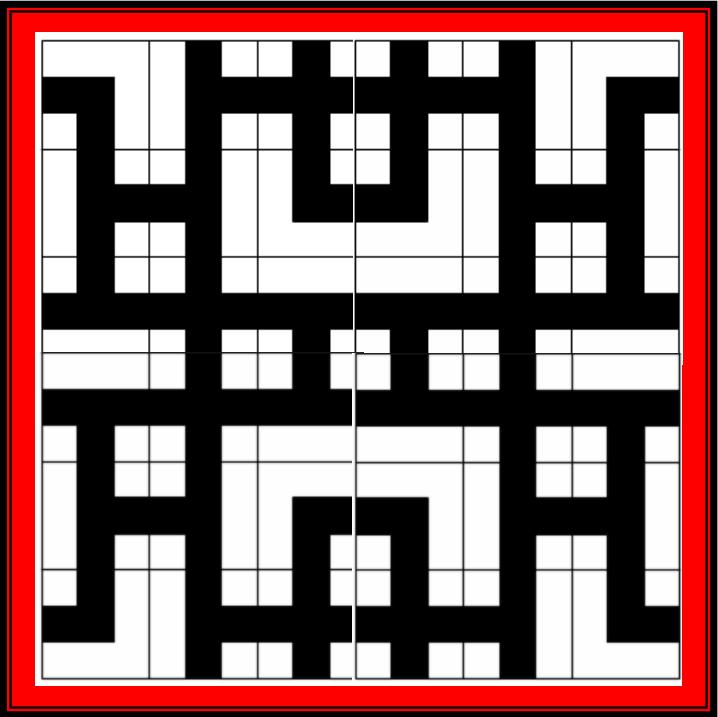
A horizontal reflection



A vertical reflection

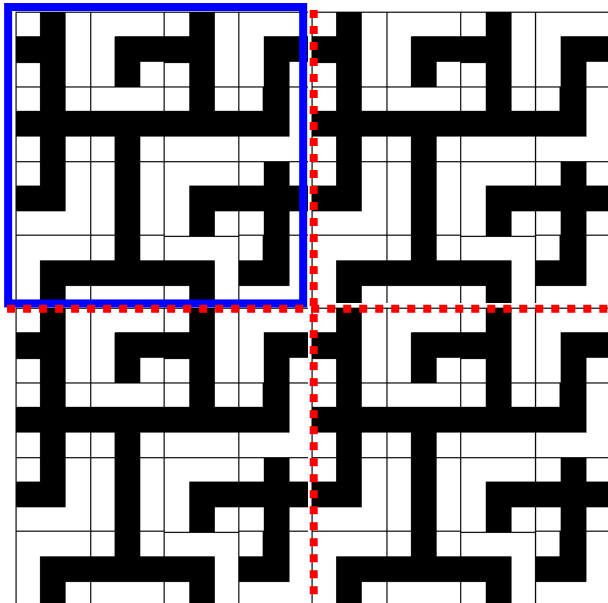


Either a vertical reflection of B or a horizontal reflection of C

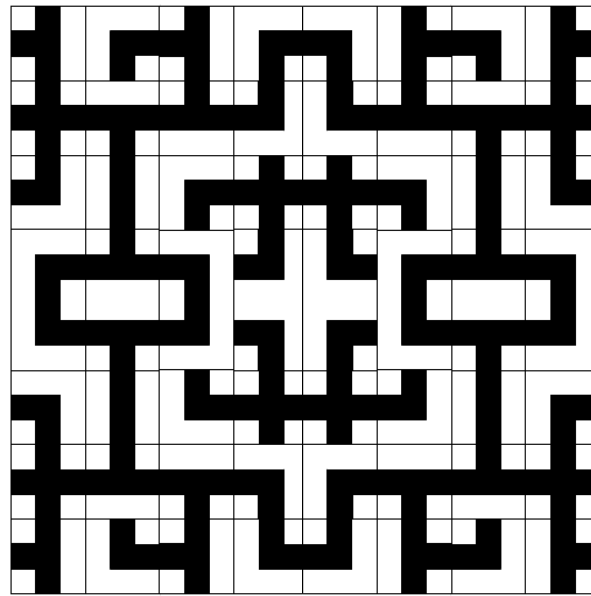


The 2 by 2 pattern extended further

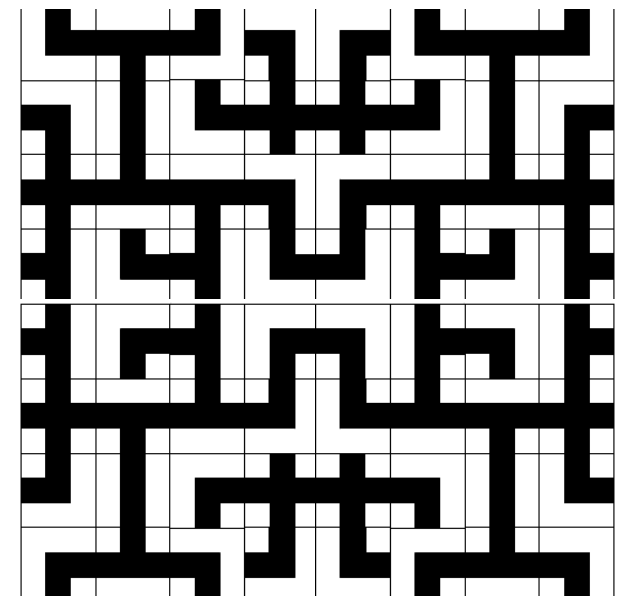
# CREATING PATTERN 2 – Translating, Reflecting, Rotating



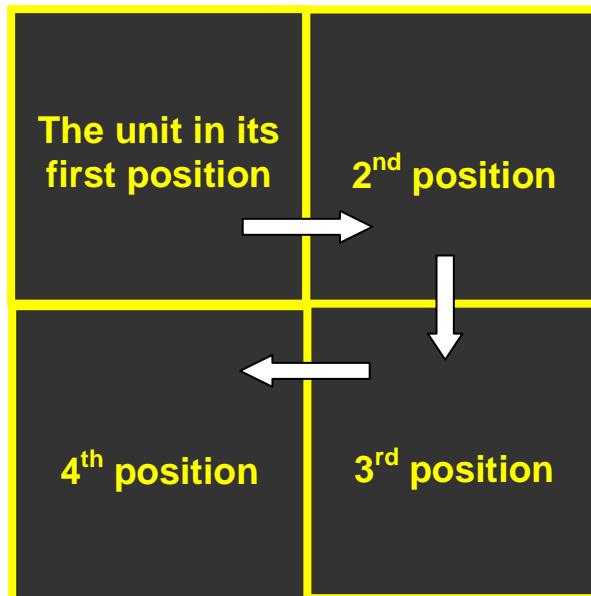
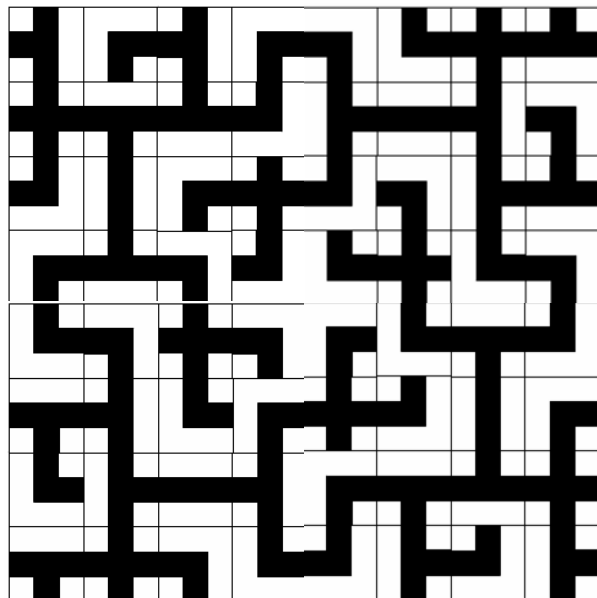
A



B



C



4 patterns made from the same unit of 16 tiles

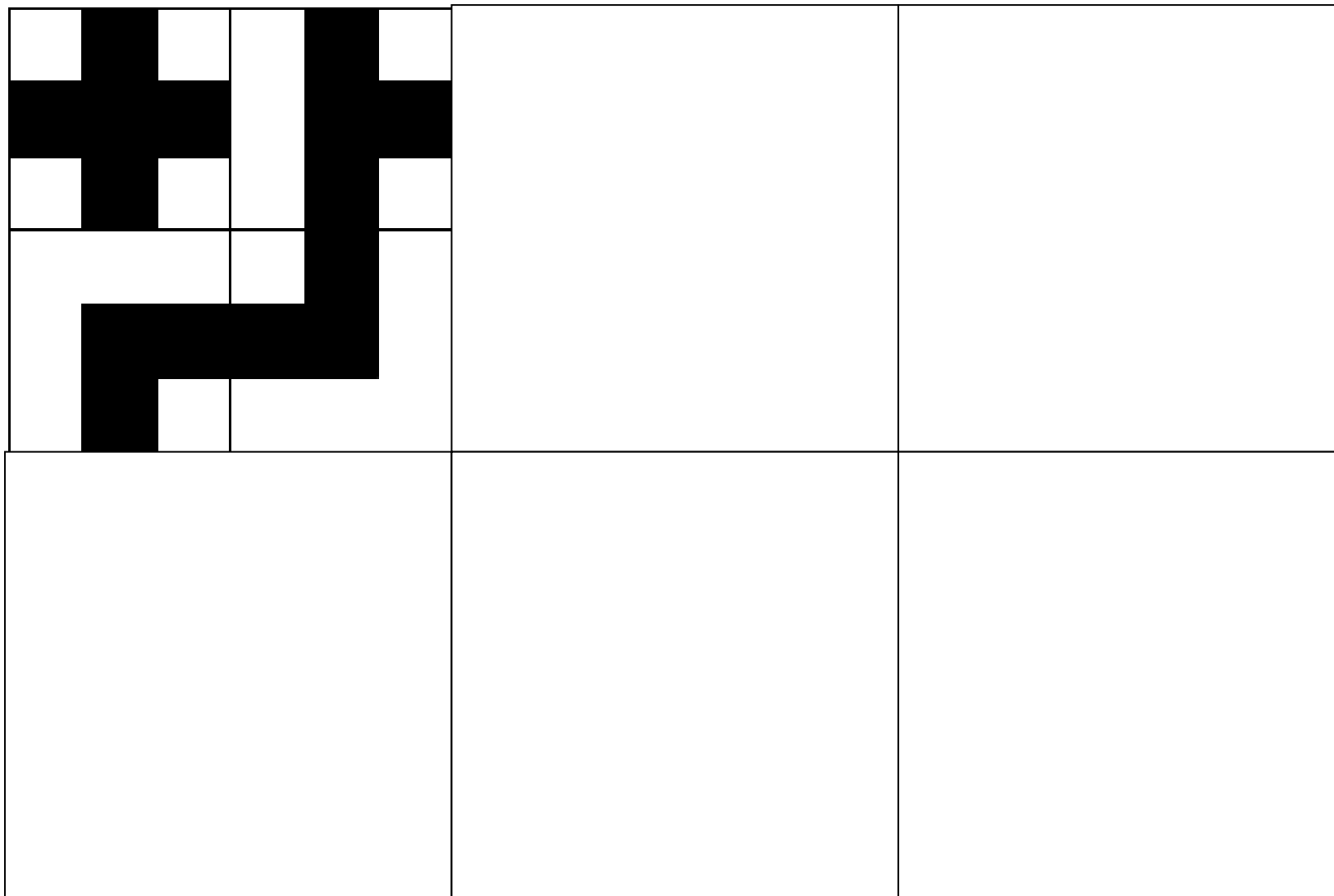
- A** The unit (top left) has been **translated** to create this pattern
- B** The unit has been **reflected horizontally** then **vertically**
- C** Same as **B**, but the unit was first rotated.
- D** The unit has been rotated 90° with each new placement going clockwise.

# Activity sheet 1

Treat these 4 tiles as a ONE TILE unit.

Reflect it in each of the lines.

Use the card tiles or stick down paper copies

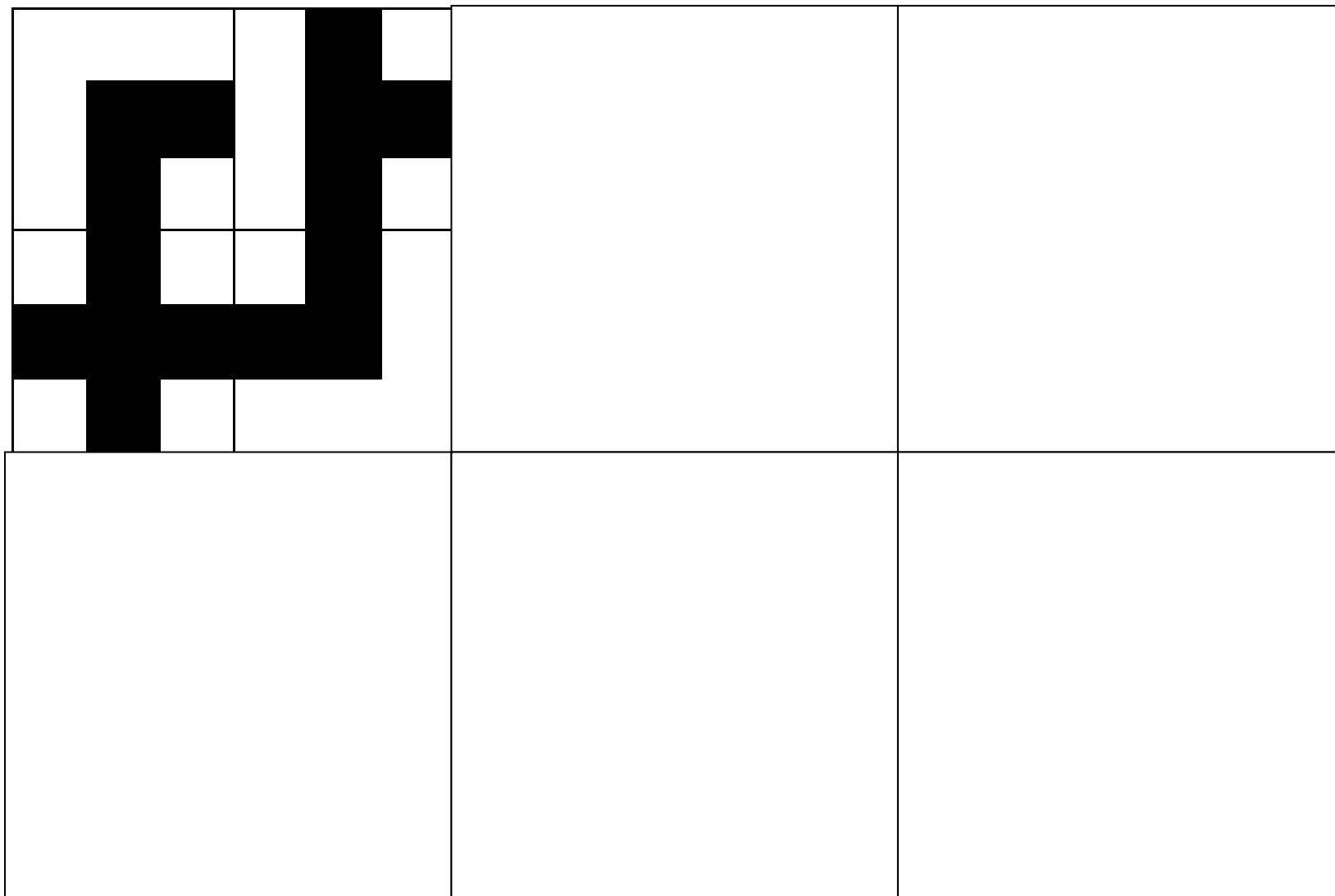


## Activity sheet 2

Treat these 4 tiles as a ONE TILE unit.

Reflect it in each of the lines.

Use the card tiles or stick down paper copies

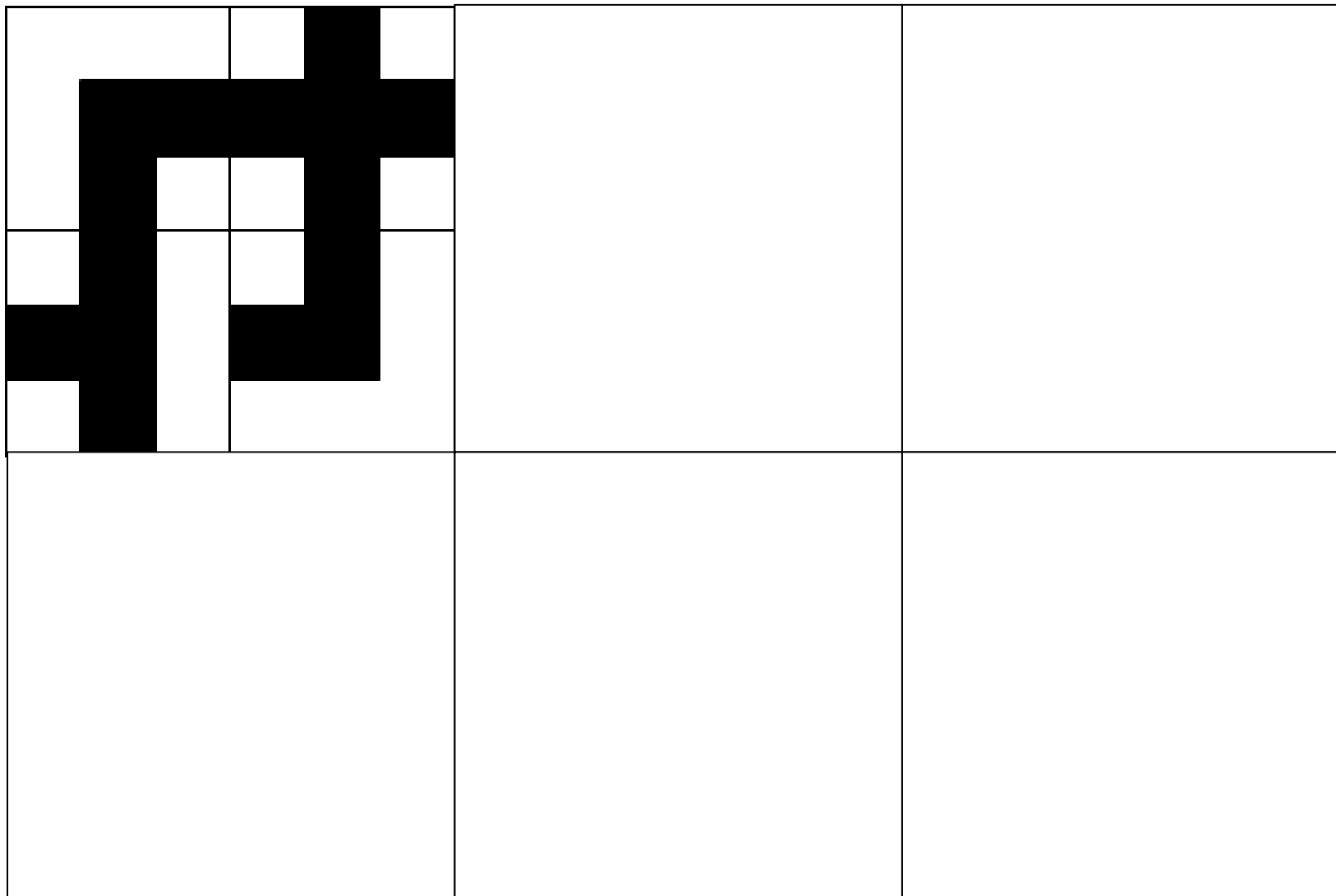


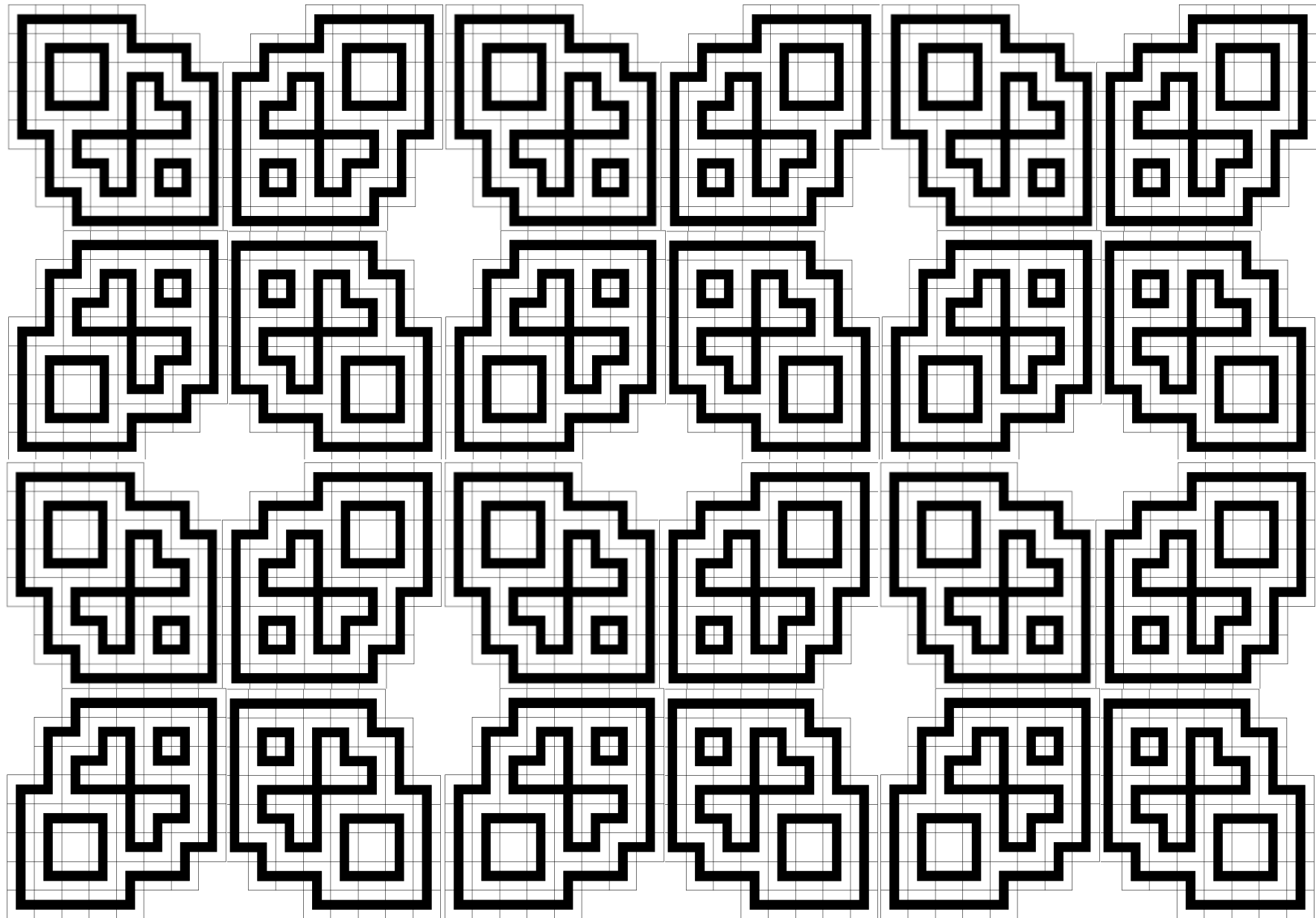
# Activity sheet 3

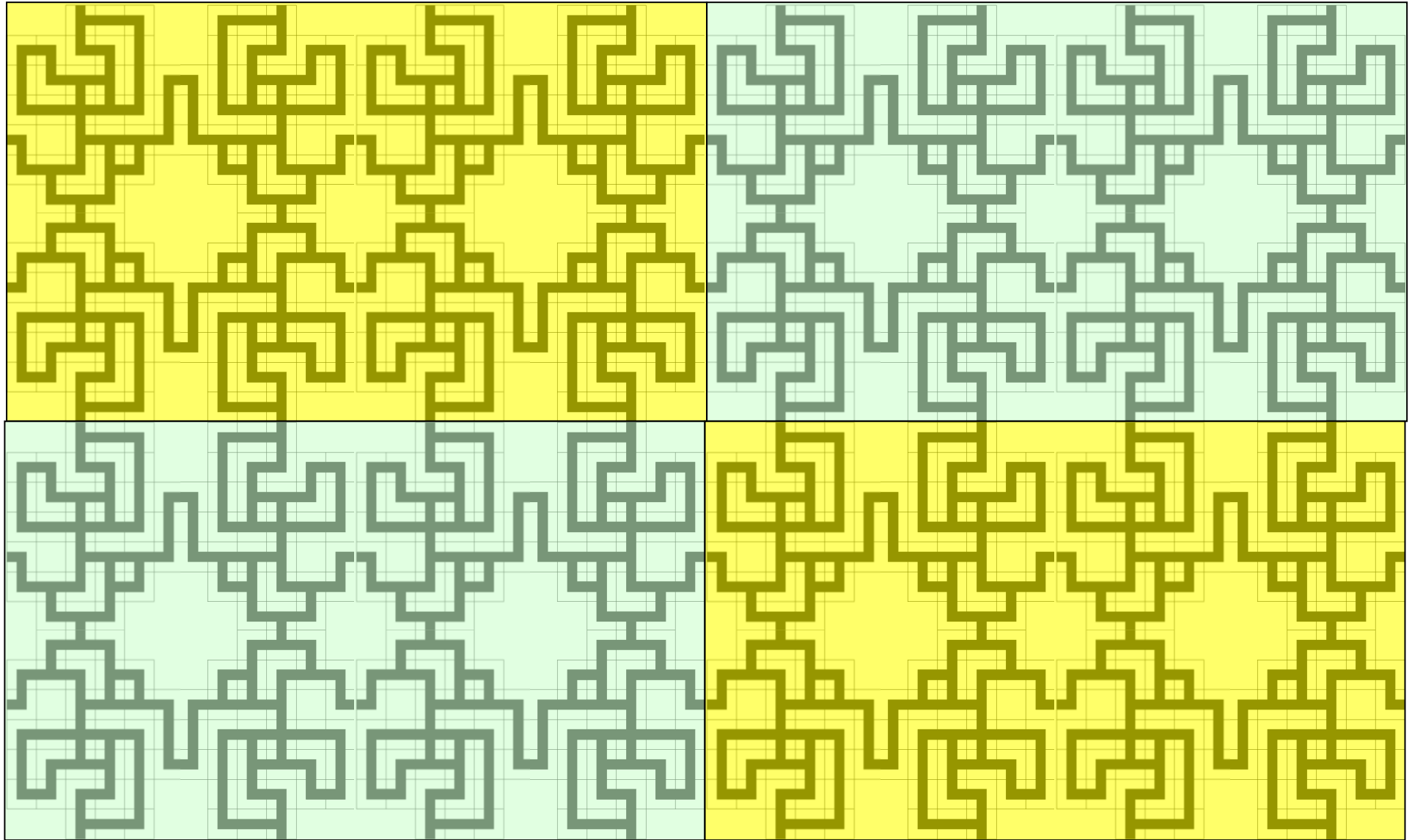
Treat these 4 tiles as a ONE TILE unit.

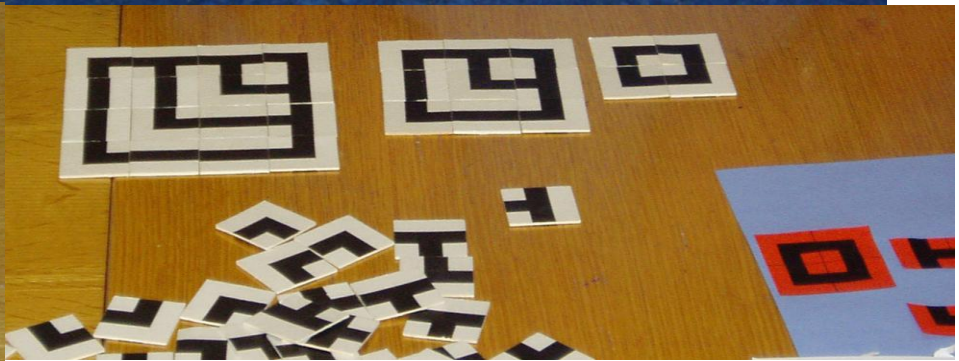
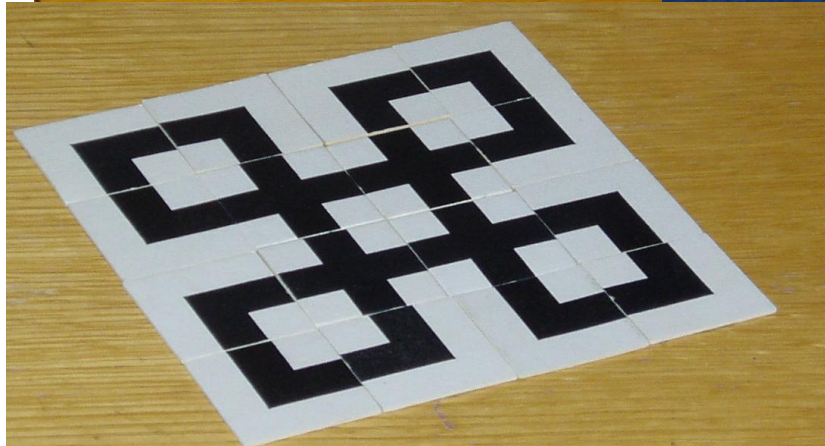
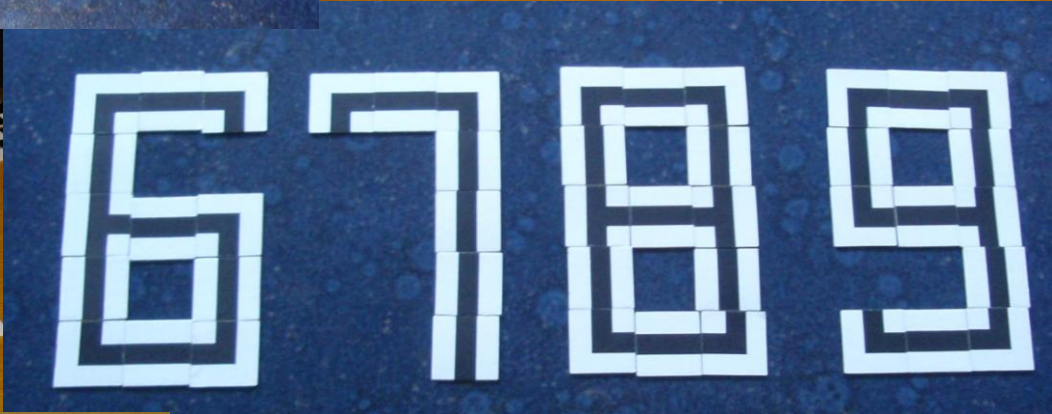
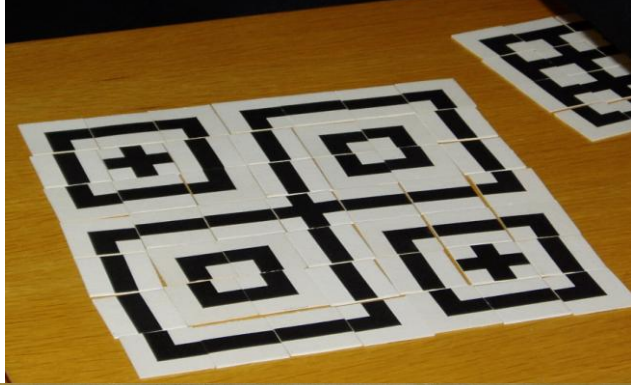
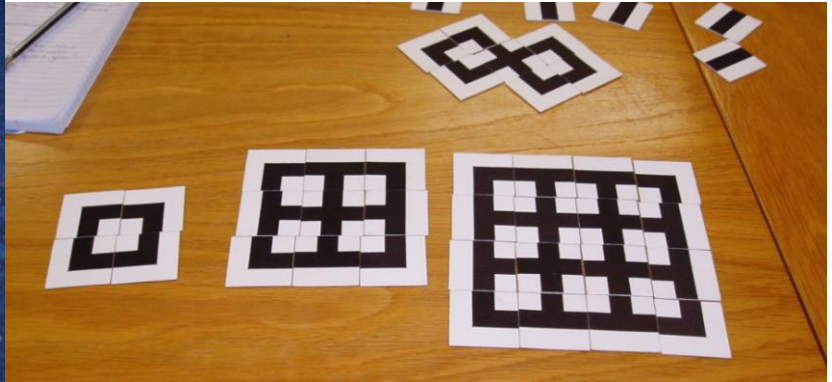
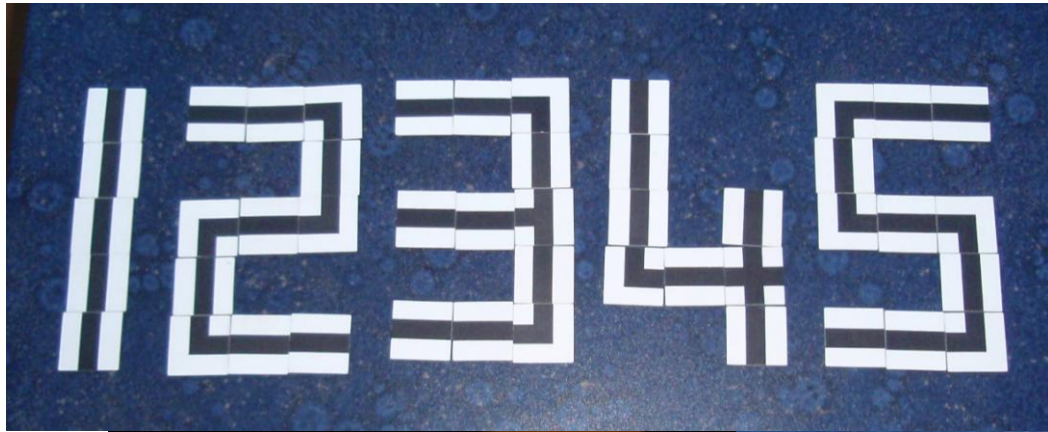
Reflect it in each of the lines.

Use the card tiles or stick down paper copies





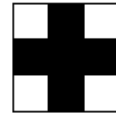
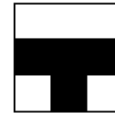
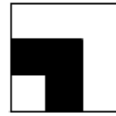






**Total**

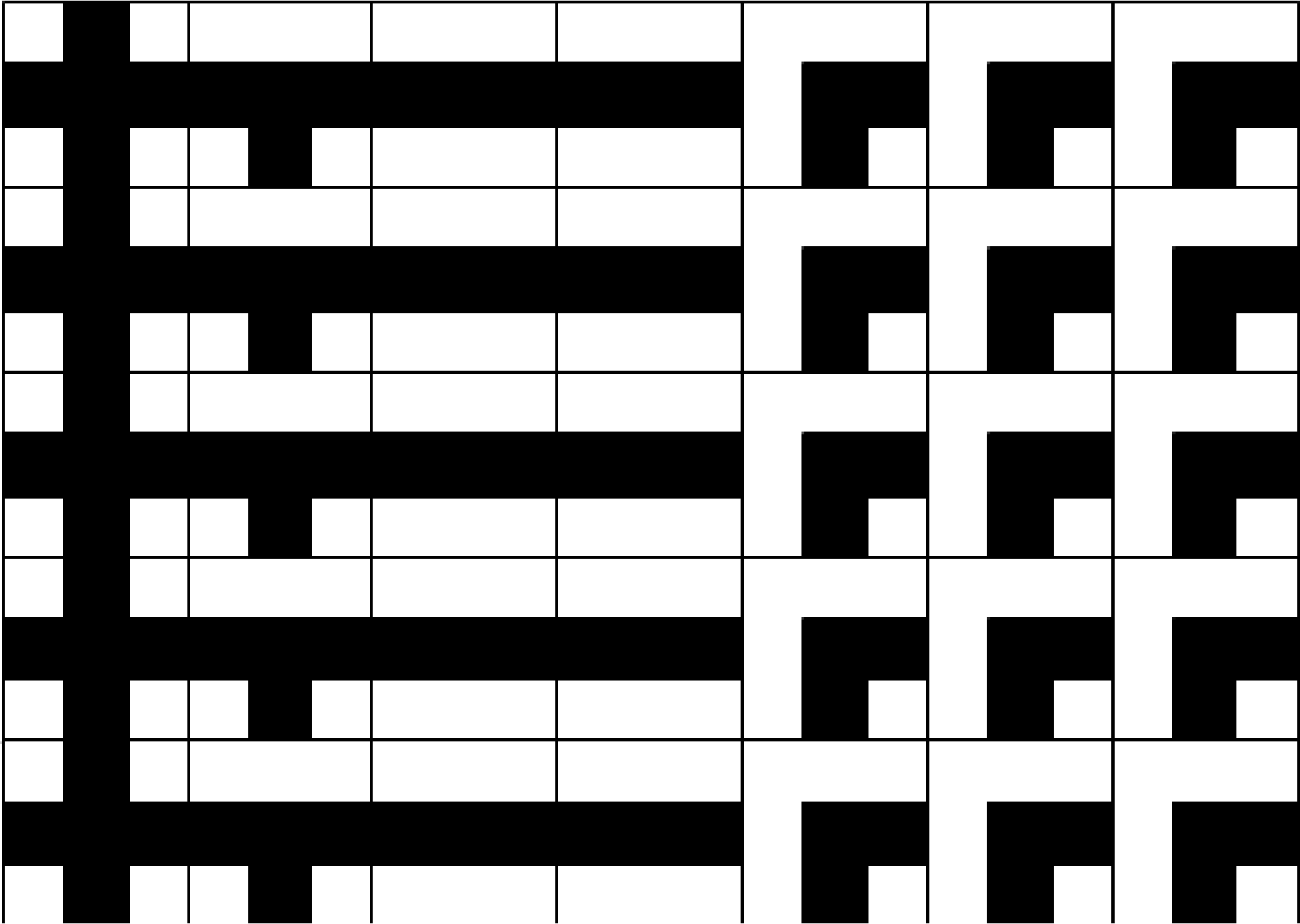
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					



**Total**

1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Actual size (36mm)



Smaller size (25mm)

